Linear Functions and Applications

Let y be a variable whose value depends on x.

Notation : $y = f(x)$ ← say y is a **function** in x.

↑ independent variable

↑ dependent variable

Example : $y = f(x) = mx + b$ (linear function)

Supply and Demand

A product is to be produced and sold

Supply = Relationship between p, the unit price, and q, the number of units a supplier is willing to provide.

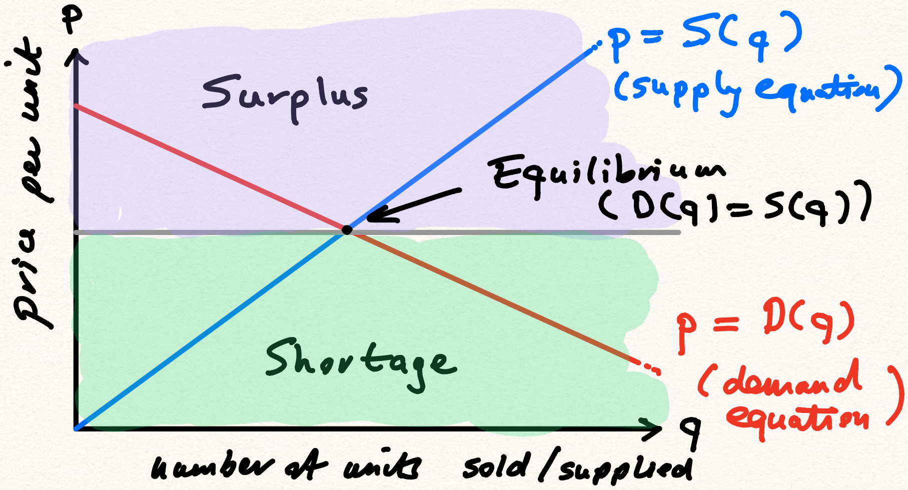
Law of Supply : As <u>p increases, q increases</u>.

Demand = Relationship between p, the unit price, and q, the number of units that will sell.

Law of Demand : As <u>p increases, q decreases</u>.

Simplifying Assumption : Supply and Demand are linear.

Picture :



Remark : It would be more sensible to have p as the independent variable. Economists are crazy and started doing this in 1920s. Using algebra we can change this. E.g.

$$p = D(q) = 20 - 2q \Rightarrow q = 10 - \tfrac{1}{2}p.$$

Examples

1/ If $S(q) = 40q$, what must the price per unit be to ensure 6 units are supplied? $p = S(q)$. $q = 6 \Rightarrow p = S(6) = 40 \times 6 = \$240$.

2/ If $D(q) = 100 - 2q$ and $S(q) = 8q$. At what price will the equilibrium be?
$D(q) = S(q) \Rightarrow 100 - 2q = 8q \Rightarrow 100 = 10q$
$\Rightarrow q = 10 \Rightarrow p = D(10) = S(10) = \$80.$

Cost, Revenue and Profit

$C(x)$ = cost of producing x units of product

$R(x)$ = revenue from selling x units

$P(x)$ = profit from producing and selling x units

$P(x) = R(x) - C(x)$

$R(x) = px$ (p = price per unit)

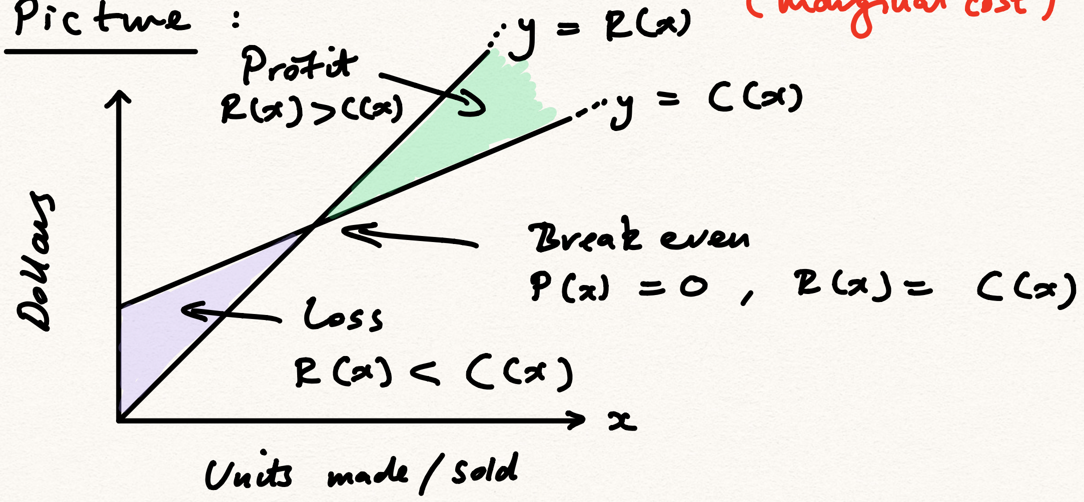
If p is fixed $\Rightarrow$ $R(x)$ linear

Simplifying assumption : $C(x) = mx + b$

$m > 0$ is extra cost of making one more unit (marginal cost)

Flat costs. E.g. factory rental

Picture :



Example $p = 20$, $R(x) = 20x$, $C(x) = 90 + 7x$

How much must they make and sell to make a profit. $R(x) = C(x) \Rightarrow 20x = 90 + 7x$

$\Rightarrow 13x = 90 \Rightarrow x = \frac{90}{13}$. They must make and sell at least $\frac{90}{13}$ units to make profit.